MP3, Grade 5

Task: Filling Boxes

Practice standard focus: Mathematically proficient students at the elementary grades construct mathematical arguments—that is, explain the reasoning underlying a strategy, solution, or conjecture—using concrete referents such as objects, drawings, diagrams, and actions. . . . Arguments may also rely on definitions, previously established results, properties, or structures. . . . Mathematically proficient students present their arguments in the form of representations, actions on those representations, and explanations in words (oral or written). In the elementary grades, arguments are often a combination of all three. Some of their arguments apply to individual problems, but others are about conjectures based on regularities they have noticed across multiple problems.

Content standard focus: 5.MD Understand concepts of volume and relate volume to multiplication and addition.

Introduction

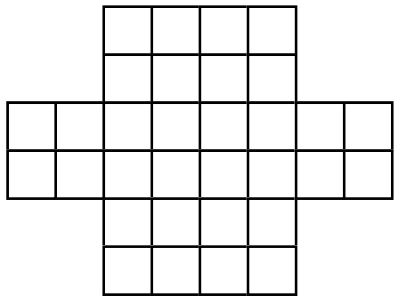
Understanding the volume of a rectangular solid is not simply a matter of memorizing and applying the formula $l \times w \times h$. Rather, students must be able to view rectangular prisms as decomposed into layers of arrays of cubes and derive their strategy for finding volume from such a decomposition.

In the following account from a fifth-grade teacher, three students present arguments to explain their strategies for finding the volume of a particular box. Two of the arguments are based on different strategies for decomposing the box into layers of arrays. The third argument is based on flawed reasoning.

Classroom example[1]

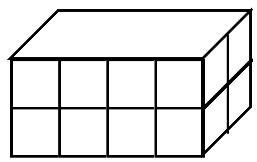
I was working with fifth graders from a unit in which students develop strategies for determining the number of cubes needed to fill paper boxes. The first activity was to look at a box net (a pattern) which could be folded up to create an open box. The children were to predict how many cubes were needed to fill the box.

[1] Adapted and reprinted with permission from Schifter, D., Bastable, V., & Russell, S.J., *Developing Mathematical Ideas: Measuring Space in One, Two, and Three Dimensions Casebook*, pp. 158-161. Copyright 2008 by the Educational Development Center, Inc. Published by Pearson Education, Inc. All rights reserved. Pseudonyms are used for teacher and students.

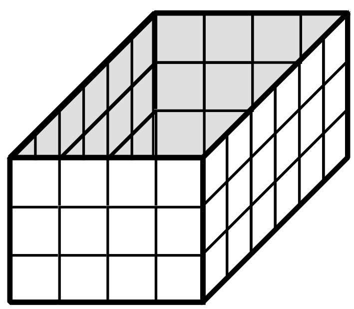


How many cubes fill this box?

Then, to verify their predictions, students copied the net onto graph paper, cut it out, folded it, taped it together to form a box, and filled it with cubes.



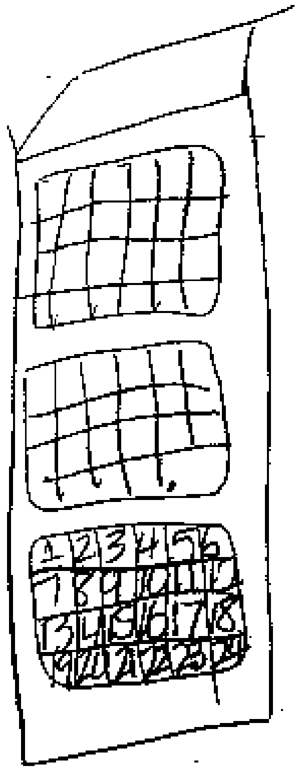
After several days of activity, I displayed a box and asked the class to show on paper, with pictures, words, and numbers, their strategy for determining the number of cubes needed to fill a 6 × 4 × 3 box. I also asked the students to relate their arguments orally.



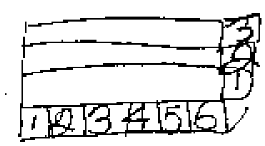
Here are three of the students' arguments.

Efren's written explanation reads: The first thing I do is count the bottom squares (24) and then I count how high the walls are (3) and then I times × the number of squares on the bottom by the number of the hight (sic) of the walls and I get my answers. If on the bottom of the box there are 24 squares on the bottom and three squares in hight I must × 24 × 3 and I get 72 as an answer.

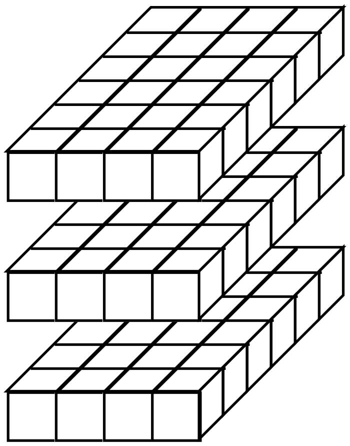
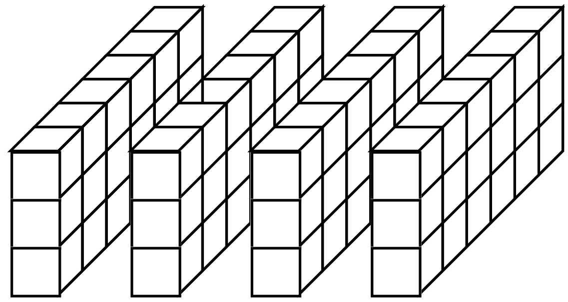
Here is Efren's picture:



Orlando: I looked at the bottom of the box, too, and I counted—1, 2, 3, 4, 5, 6—and when I got to the end I kept on counting up the side—1, 2, 3. So I said three times six is eighteen. Then I looked at the bottom again and counted four across, so I knew that I need four groups of eighteen. The answer is 72.



Both boys saw the cubes in layers inside the box. Efren's layers are horizontal: "flat layers" or "like slabs one on top of the other," he calls them. Orlando's are vertical, as depicted in the following diagram.

Leo: I counted the bottom, the rows and the columns, and got 24. Then I counted the ends, 3 rows and 4 columns. I got 12. Then I counted the sides, 3 rows and 6 columns. I added 24 and 12 and 18 and all together there are 54 squares in the whole box.

I noticed that he said “sides” and “ends,” using the plural, but he clearly counted only one side (12) and one end (18). Although his plan was incorrect, he didn’t even follow it in this instance. Overall, I wondered how Leo was looking at this box. I didn’t think he realized that this three-dimensional box was more than just two-dimensional rectangles taped together. He didn’t seem to have the conceptual understanding that when a two-dimensional pattern is cut out and taped together to form a three-dimensional shape, some new entity is created; it has a new attribute. The box now has volume. I thought Leo had a view that one cube was sitting on each square of the pattern. When the pattern was cut and taped together into a box, the cubes were now on the inside of the box filling it completely. He didn’t see that there would be some space on the inside of the box that had not existed beforehand.

My next step is to help Leo think about the inside of the box. What will happen if we try to fill the empty box with the number of cubes he predicted in the exact way he counted them? Will this help him visualize?

Commentary

In this example, three students present arguments to justify their solution for the volume of a 6 × 4 × 3 box. Their arguments are based on the structure of the rectangular prism—how it can be viewed as made up of layers of arrays. They focus on the volume of a particular box, but Efren also uses more general language that indicates he is thinking about the regularity of how any box shaped like a rectangular prism is structured: “I times the number of squares on the bottom by the number of the hight (sic) of the walls.”. The students use objects and drawings in their arguments, as well as words and actions to describe how they see the layers of the cubes in the box.

Once students offer their arguments, part of the task of the teacher is to assess the arguments to determine the soundness of their students’ reasoning. It is important that the classroom culture both affirms sound reasoning and also respects the reasoning of students even when it is incorrect—there is often much to be learned from considering why incorrect reasoning may appear, at first, to make sense and why it does not work to solve the problem. In this example, the teacher thinks about how to help Leo consider why the way he is counting squares does not result in the volume of the box; she hypothesizes that the student isn't yet visualizing the interior of the box and plans to set up a task to help him see why his strategy doesn't work.

Note that, after having worked on finding the volume of several boxes of different dimensions, as Efren explains his strategy for finding the volume of the $6 \times 4 \times 3$ box, he presents a general method: count the bottom squares and then count how high the walls are, and then multiply the number of squares on the bottom by the number of the height of the walls. This is an illustration of MP8: *Look for and express regularity in repeated reasoning*.