MP8, Grade 5

Task: Today's Number

Practice standard focus: MP8. Mathematically proficient students at the elementary grades look for regularities as they solve multiple related problems, then identify and describe these regularities.

Content standard focus: 5.OA. Write and interpret numerical expressions. Analyze patterns and relationships. 5.NBT. Perform operations with multi-digit whole numbers (in particular, the focus on properties of operations).

Introduction

There are many variations of a task that some teachers call "Number of the Day" or "Today's Number." Typically, teachers provide a target number, and students are challenged to write expressions equivalent to that number. By making strategic choices of both the target number and constraints for the expressions students are to write, teachers can focus on particular mathematics content. Teachers can also ask students to find patterns in their expressions, and to describe and account for the regularities they see in these patterns.

Depending on the constraints teachers choose, this task can be used at all grade levels. For example, first graders might be asked to find addition expressions equal to 10, and they might generate this pattern:

0 + 10
1 + 9
2 + 8
3 + 7
.
.
.

This could lead to a discussion about how to change one addition expression to another while maintaining the same sum. [See the grade 1 example for MP8, Equivalent Addition Problems.]

Today's Number can also be used in older grades to generate examination of other kinds of numbers (e.g., fractions) and other operations. In the following example, students generate ideas about how to create equivalent division expressions.

Classroom Example

The following is a part of a real account from a fifth-grade teacher. Pseudonyms are used for students[1]*:*

For Today's Number, I chose 24 as the target number because I am going to introduce a constraint that involves division, and I know that all of my students, including my struggling learners, have access to multiples of 24. I give them just a few minutes to write down some expressions using any operation before I pull them together. I select several students to share division expressions—24÷1, 48÷2, 72÷3—and then I focus the class on Salena's equation: 48 ÷ 2 = 24**.**

Teacher: How did you come up with that equation?

Salena: Well, I thought about two 24s and that would be 48 so if I divided 48 in half, I would get 24.

Salena struggled with math ideas at the beginning of the school year. I came to realize that she had some strong numeric reasoning that was buried under the need to use strategies she didn't really understand. I am glad to be able to use her idea to begin our work with equivalent division expressions.

Teacher: I would like each of you to work for a few more minutes independently. Write some other division expressions that would equal 24 in your journals.

Throughout the school year, we have periodically used Today's Number to examine how to create equivalent expressions using different operations. For example, when we studied multiplication, the class devised the following conjecture: "When you try to get the same product but change the factors, you must multiply a factor by a certain number, and divide the other factor by the same number." After the class agreed on the conjecture, we worked together to come up with story contexts and representations to show that it has to be true.

My students understand that now we are moving toward a similar examination of division. As I rotate around the room, I see the following sets of expressions:

Salena:

48÷2
72÷3
96÷4
120÷5
144÷6

[1] Unpublished case from the project *Foundations of Algebra in the Elementary and Middle Grades*. Funded in part by the National Science Foundation. © TERC 2009.

I notice that Salena added one more group of 24 in each row.

Eddie:

$96 \div 4$
$192 \div 8$
$384 \div 16$
$768 \div 32$

Eddie repeatedly doubled both the dividend and the divisor.

Sean:

$240 \div 10$
$480 \div 20$
$720 \div 30$
$2400 \div 100$
$4800 \div 200$
$7200 \div 300$
$24000 \div 1000$

After the class listed $24 \div 1$, $48 \div 2$, $72 \div 3$, Sean created equivalent expressions by multiplying the dividend and divisor by powers of ten.

Anna:

> You find multiples of 24 and then you divide it by how many 24s are in it and you'll get 24. So to get it you do this equation: $24 \times n \div n = 24$

Anna produced only three expressions in her journal before she wrote this generalization.

Amelia:

$Y \div 1 = Y$
$Y \times 2 \div 2 = Y$
$Y \times 3 \div 3 = Y$

> Every time you double it, it will take the number you double and divide it by the number you double it, canceling it out.

I think Amelia was trying to express that the action of increasing the dividend and divisor in the same way was in essence doing nothing to the quotient ("canceling out" the action).

I am impressed with the range of responses. All of the students entered the task and there are a variety of ideas with which we can continue this work.

It is almost time to end math workshop so I decide to leave the students thinking about the expressions they have created. I say, "Everybody has produced a systematic list or written a statement. My question is, do you think what you have done would work no matter what number is chosen for Today's Number?"

Cole: I think it could because we are just changing things around like we did with addition.

Kathryn: But we increase or decrease both of the numbers. We aren't moving pieces around, like in addition.

Amelia: But we always can get the same answer.

Will: And it's different than multiplication because we change both numbers in the same way.

Brent: We could be here forever, because you can just keep making new ones!

Teacher: Why do you think we can keep making new ones forever?

Amelia: Because we aren't really changing the answer. We keep just changing the number of groups and how many we had.

Teacher: Well, these are some great ideas to work on next time. Let's keep thinking about a few things: How is it that we are changing the numbers and still getting the same quotient? What stories or representations could we make to show that? And how is division different from the other operations?

I am anxious to continue this conversation and to see what the rest of the class might think of Anna's and Amelia's work. Will Anna and Amelia be able to represent their ideas with a story or a model?

Commentary

In this example, the teacher uses Today's Number to give students an experience of MP8: *Look for and express regularity in repeated reasoning*. The regularity the class is examining is how the dividend and divisor change to create expressions with the same quotient.

The class has already studied equivalent addition expressions, subtraction expressions, and multiplication expressions. Knowing where they are headed, the students create division expressions with intention. Some students have created lists of expressions, selecting expressions in a systematic way. Anna has articulated a general strategy for finding any division expression equal to 24. Amelia is working to articulate a strategy for finding equivalent division expressions equal to any number, Y.

The teacher indicates that once students articulate a generalization about equivalent division problems, the work will not be done. She has asked them to think about stories and representations they could use to explain *why* division behaves this way. That is, the activity is not simply to develop more rules about how to manipulate symbols, but to keep in mind the actions of the operations. In this way, mathematics remains an activity of sense making.

Note that the numbers the students are using in this example are smaller than the four-digit dividends and two-digit divisors fifth graders are expected to be able to compute. It is important to recognize that this is not a lesson in computation. Rather, this lesson is part of a larger discussion of the behavior of division. Students are working to articulate generalizations that apply to *all* numbers, small and large.