MP3, Grade 2

Task: How Do You Know That 23 + 2 = 2 + 23?

Practice standard focus: MP3. Construct viable arguments and critique the reasoning of others. Mathematically proficient students at the elementary grades construct mathematical arguments—that is, explain the reasoning underlying a strategy, solution, or conjecture—using concrete referents such as objects, drawings, diagrams, and actions. . . . Mathematically proficient students present their arguments in the form of representations, actions on those representations, and explanations in words (oral or written). In the elementary grades, arguments are often a combination of all three. Some of their arguments apply to individual problems, but others are about conjectures based on regularities they have noticed across multiple problems. . . . Students might also use counterexamples to argue that a conjecture is not true . . . .

Content standard focus: NBT.B.5. Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. NBT.B.7. Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction

Introduction

In this example, many of the students are focused on the regularity they notice: that addition is commutative no matter what numbers they use, and subtraction is not. However, in the course of their discussion, students are also offering arguments about why the commutative property of addition holds. This commentary focuses on how these young students develop their arguments.

In this second grade classroom, the teacher has noticed that the idea that addition is commutative has come up in her students' work and talk. She decides to structure an opportunity for students to talk explicitly about this idea. She asks her students to create addition expressions that sum to 25. As the teacher records students' expressions, inevitably pairs of expressions are suggested in which the order of the two addends is reversed.

19+6=25	6+19=25
15+10=25	
20+5=25	
12+13=25	
5+10+10=25	
18+7=25	7+18=25
0+25=25	25+0=25
23+2=25	

The teacher brings her students' attention to the question: What happens to the sum if the order of the addends is changed?

The video segment[1] begins as the teacher poses the question, "Can we change the order [of the addends] and still get 25?"

Video: How Do You Know That 23 + 2 = 2 + 23? http://vimeo.com/66205198

Commentary

This commentary focuses particularly on the students' construction of arguments. (For further commentary on this illustration, focusing on MP8, see the version of this example under MP8.)

At 1:58, Ashanti says that 19 + 6 will have the same sum as 6 + 19 "because you're just switching the numbers around. You're not adding any more numbers or taking away any numbers." The teacher then draws students' attention to a representation of 2 + 23, using connecting cubes. The students seem quite sure that you can always change the order of addends without changing the sum. After all, they have been working with addition and what it means since kindergarten. It is a familiar operation and switching the order of addends is a familiar idea. However, the teacher is asking them to think more deeply about this idea and to show *why* it is true: sometimes it is exactly those ideas that seem most "obvious" that are most difficult to explain with a convincing argument.

[1] From *Reasoning Algebraically About Operations* by Deborah Schifter, Virginia Bastable, and Susan Jo Russell. DVD, Session 3, "How Do You Know That 23 + 2 = 2 + 23? Grade 2."

By manipulating the cubes, Amira shows that changing the order of the addends does not change the sum. There are three important characteristics of her representation. First, it shows the action of the operation—there is an implied joining of the two stick of cubes. Second, the action, combined with her explanation, shows how the conclusion (that the sum does not change) follows from the action of changing the order of the addends: as she explains, "you're not taking away or adding nothing to it . . . it would still be the same number."

Finally, there is the potential to use this representation to talk about any numbers (or, at least, any positive, whole numbers—the domain these students are considering). While Amira and other students may not yet be thinking how Amira's representation can accommodate other numbers, it is possible to imagine that the bunches of cubes represent any two amounts that are combined first in one order, then in another. The argument about why the sum remains the same does not depend on knowing what particular amounts the bunches of cubes represent. The way that Amira manipulates the cubes without continuing to refer specifically to the original problem (2 + 23) may indicate that she is already thinking about this representation in a more general way.

When the teacher asks students to consider a subtraction problem, 7 - 3, they come up with further arguments—this time about why subtraction is *not* commutative. When students first study properties and behaviors of the operations, they may at first think that the property applies to the *numbers* rather than to a specific operation. By comparing the action of changing the order of the numbers in both addition and subtraction expressions, the students can formulate images of how addition and subtraction each work, how they are related, and how they behave differently. In this discussion, the teacher is not looking for a correct solution for 3 – 7; second graders are not expected to have the knowledge of negative numbers needed to solve this problem (although Laguar does have some knowledge of negative numbers). Based on what they do know in the domain of positive whole numbers and 0, Amira and Ashanti come up with reasonable arguments about how subtraction behaves differently from addition. Amira thinks about removing 7 when there are only 3 things; Ashanti thinks about starting at 3 and counting back 7:

Amira: That if you have 3 take away 7, but 3 doesn't have 7, so you can only do 7 and 3 cause you don't, cause 3 is not a 7 . . . You could only take away 3 to make zero . . .

Ashanti: I think that you can't use the 3 because after you use the 3, 3, 2, 1, 0, 0, 0, it's going to keep on repeating itself until it gets to 7 . . .

They can see that changing the order of the 7 and 3 *does* change the result of the subtraction. In fact, they have found a counterexample, showing when commutativity does *not* work, that helps them define when it does.

Representations are elementary students' primary basis for constructing arguments. By using such a representation-based argument, students can show the action of the operation, accommodate a class of instances (e.g., all positive whole numbers), and demonstrate how their conclusion (in this case, that the sum is not changed) follows from their premise (in this case, that the order of the addends is changed). Unlike formal proofs, which are often captured entirely with symbols, elementary students' proofs combine representations, actions on those representations, and words that describe their representations and actions.

Such discussions about the properties of operations encourage students to think about an operation as a mathematical object in itself. An operation, such as addition, is not simply a set of procedures for solving problems; it is an object of study that has mathematical characteristics. Understanding that addition is essentially different from (although related to) subtraction, or that addition is essentially different from (although related to) multiplication is important for students' work in the elementary grades and in later study of more advanced mathematics.

In this class session, many of the students are engaging in MP8, *Look for and express regularity in repeated reasoning*, but some students are beginning to construct arguments (MP3). More students could now work on constructing arguments for changing the order of two addends, and they might go on to consider what happens when more than two addends are rearranged. See the illustration, "How Do You Know That 23 + 2 = 2 + 23?" under MP8 for more about how this episode illustrates that practice; see: http://www.illustrativemathematics.org/static/practice_standards/MP8_Grade2_WhyDoes23+2_withvideo.pdf.