MP8, Grade 2

Task: How Do You Know That 23 + 2 = 2 + 23?

Practice standard focus: MP8. Mathematically proficient students at the elementary grades look for regularities as they solve multiple related problems, then identify and describe these regularities. . . . Mathematically proficient students formulate conjectures about what they notice . . . .

Content standard focus: NBT.B.5. Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. NBT.B.7. Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction

Introduction

What does it mean to use "properties of operations" in the primary grades? The focus is not learning names for the properties, but on understanding how each operation behaves—for example, that the numbers in an addition expression can be rearranged without changing the sum. In this second grade classroom, the teacher has noticed that the idea that addition is commutative has come up in her students' work and talk. She decides to structure an opportunity for students to talk explicitly about this idea. She asks her students to create addition expressions that sum to 25. As the teacher records students' expressions, inevitably pairs of expressions are suggested in which the order of the two addends is reversed.

19+6=25	6+19=25
15+10=25	
20+5=25	
12+13=25	
5+10+10=25	
18+7=25	7+18=25
0+25=25	25+0=25
23+2=25	

The video segment[1] begins as the teacher poses the question, "Can we change the order [of the addends] and still get 25?"

Video: How Do You Know That 23 + 2 = 2 + 23? http://vimeo.com/66205198

Commentary

These second graders have frequently noticed the regularity that if you change the order of two addends the sum remains the same. The teacher might simply assume that because they've noticed this property, they understand and can apply it and extend it. However, investigating a property of an operation, even if that property seems intuitively obvious to many students, is mathematically important for many reasons. First, such a discussion slows students down to focus attention on articulating carefully what the regularity is that they are noticing. Second, students have the opportunity to consider for what numbers this regularity holds. When they consider expressions like 23 + 2 and 2 + 23, many of the students "just know" that the answers are the same. By posing an addition problem with numbers that are large enough so that students don't just know the answers, the teacher opens up the opportunity for students to think about the action of addition and why it makes sense that, no matter the size of the numbers, the sum does not change when the order of the addends changes.

This focus on the action of the operation, which leads to considering why this property holds for addition, is extended when the teacher asks students what happens for the subtraction expression, 7 – 3. When students first study properties and behaviors of the operations, they may at first think that the property applies to the *numbers* rather than to a specific operation. By comparing the action of changing the order of the numbers in both addition and subtraction expressions, the students can formulate images of how addition and subtraction each work, how they are related, and how they behave differently. In this discussion, the teacher is not looking for a correct solution for 3 – 7; second graders are not expected to have the knowledge of negative numbers needed to solve this problem (although Laguar does have some knowledge of negative numbers). Both Amira and Ashanti come up with reasonable explanations about how subtraction behaves differently from addition. Even though their knowledge of the number system includes only positive whole numbers and zero, they can see that changing the order of the 7 and 3 does change the result of the subtraction. They are engaged in important reasoning about the difference between addition and subtraction.

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[1] From *Reasoning Algebraically About Operations* by Deborah Schifter, Virginia Bastable, and Susan Jo Russell. DVD, Session 3, "How Do You Know That 23 + 2 = 2 + 23? Grade 2."

Finally, such discussions about the properties of operations encourage students to think about an operation as a mathematical object in itself. An operation, such as addition, is not simply a set of procedures for solving problems; it is an object of study that has mathematical characteristics. Understanding that addition is essentially different from (although related to) subtraction, or that addition is essentially different from (although related to) multiplication is important for students' work in the elementary grades and in later study of more advanced mathematics.

This discussion is an opening for further study of addition. In this class session, most of the students in the class are engaging in MP8, *Look for and express regularity in repeated reasoning*, but some students are beginning to construct arguments (MP3). More students could now work on constructing arguments for changing the order of two addends, and they might go on to consider what happens when more than two addends are rearranged. See the illustration, "How Do You Know That 23 + 2 = 2 + 23?" under MP3, *Construct viable arguments and critique the reasoning of others*, for more about how this episode illustrates that practice http://vimeo.com/66205198.