MP8, Kindergarten

Task: Double Compare

Practice standard focus: MP8. Mathematically proficient students at the elementary grades look for regularities as they solve multiple related problems, then identify and describe these regularities.

Content standard focus: K.CC. Count to tell the number of objects. K.OA. Understand addition as putting together and adding to.

Introduction

Double Compare[1] is a card game, similar to War, but played with numeral cards, usually in pairs. For kindergarten, the cards have both a numeral from 0 to 9 and that number of objects pictured. For each round, each student pulls two cards from the deck. The students figure out the sum of the two numbers, and the student with the higher sum says "me." If the two students have the same sum, they draw an additional card.

While this game is most often used for students to engage in counting and adding, and for beginning to learn number facts, it provides the opportunity for students to notice and describe regularities about addition. In the classroom example below, the teacher notices that in some rounds of the game students add or count to find the sums on their numeral cards, while in other rounds some students do not add or count—they have other ways to reason about the two pairs of numbers. She uses what students are noticing as an opportunity to have the whole class consider regularities that occur across multiple problems, regularities that are related to the behavior of addition.

Classroom example

The following is part of a real account from a kindergarten teacher[2]:

As I watched my class play Double Compare, a situation came up with several groups: each partner would have one card equal to the other person's and one card that was different. When Martina had 6 and 2 and Karen had 6 and 1, Karen quickly said "you" [indicating that Martina had the greater sum]. I asked how she knew and she pointed to the 2 and said, "This is big. Even though these are the same [the 6s], this [the 6 and 2] must be more."

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[1] Russell, S.J., Economopoulos, K., Wittenberg, L., et al. (2008). *Investigations in Number, Data, and Space*. Second edition. Glenview, IL: Pearson.

[2] Adapted and reprinted with permission from Schifter, D., Bastable, V., & Russell, S. J. (*Reasoning Algebraically About Operations Casebook*. Pp. 31-33. Copyright © 2008 by Education Development Center, Inc. Published by Pearson Education, Inc. Used by permission. All rights reserved. The students' names are pseudonyms.

Paul and his partner had a very similar set of hands. Paul put down 6 and 3 and his partner put down 6 and 1. Paul said, "I had 6 and he had 6, and then I had a higher number." I asked what their cards added up to, and both of them counted all the little pictures on their cards to get the totals. On a turn a minute later, Paul had 4 and 3 and his partner had 4 and 5. Paul's comment was, “I have 3 and he has 5." He knew he could basically ignore the two 4s.

Another hand I saw was when Karen had 6 and 5 and Martina had 0 and 2. Karen said "Me, because she got two low numbers."

After students worked in pairs, we all gathered on the rug to talk about how students knew who got to say "me." We talked for a while about how all the pairs “ignored” cards when each partner had the same one, and only paid attention to the cards that were different. Martina said that 6 and 3 is more than 6 and 1 because the 3 is bigger than the 1. I asked, "What about the sixes?" and she said, "They're the same." Paul added, "They don't matter. You don't have to pay attention to the sixes." I pointed out to them that when I put down the 6 and 1, they said, "That's seven," but when I put down the 6 and 3, no one figured out what it made. “Would 6 and 3 make a higher number than 6 and 1?” I heard 8! 9! 10! They settled on 9 by counting all, and because Danielle said 6 plus 3 is 9. “Is 9 more than 7?” Yes!

These students seem to have made a generalization, that a number plus a big number is more than the same number plus a small number. I put out a few more sets of cards, varying the number that was the same ("Does this only work for 6?" "No.") They said it always works, and Paul reiterated that you don't have to pay attention to the numbers that are the same.

Another generalization most of them seemed to be using was that the sum of two small numbers is less than the sum of two big numbers. Karen's comment that she got two low numbers expressed this idea. I asked the group about this. I put out 1 and 5 and 0 and 4, which had been a turn in Amanda and Danielle's game. I asked how Amanda knew she had more. She said, "This [5] is bigger than this [4], and this [1] is bigger than this [0]." I asked if it would work with other numbers and everyone said yes. We tried some. They were all saying it worked. They weren't adding and counting. They were "just knowing."

Implicit in the children’s actions were two generalizations. For one, the children were close to articulating what it was: “You don’t have to pay attention to the sixes.” I wonder what it will take for them to have words for their second generalization beyond simply saying they “just knew.”

Commentary

In this episode, the teacher sees that students are noticing regularities about addition across multiple examples: First, if one number is the same in each student’s pair of cards, they need only compare the second number to determine who has the greater sum. Although these students are far from using algebraic notation, this idea can be expressed symbolically—If $a > b$, then $a + c > b + c$. Second, if the two numbers (a, b) in one pair can be matched to the two numbers in the other pair (c,d) such that $a > c$ and $b > d$, then $a + b$ is the greater sum.

The teacher realizes that, in making such observations, her students' are engaged in MP8: *Look for and express regularity in repeated reasoning*. She chooses to discuss their observations in whole group in order to make explicit the iortance of such activity. By asking them whether their idea works only for specific numbers, she opens up the opportunity for them to think about whether the regularities they are using apply to more than the particular examples they have encountered.

While the students have not yet fully articulated these ideas, this discussion is the beginning of a process of putting what they implicitly understand into words (see MP6). These general ideas about addition, emerging from the regularities they notice, are related to the meaning and properties of the operation. They might return to these generalizations in the context of other activities focused on addition and eventually construct arguments for why they occur (see MP3).